The Physical-Fourier-Amplitude Domain, and Application to Sensing Sensors

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http://veillametrics.com (Sample code available)

***Abstract*—We introduce a multidimensional space to quantify a signal or sensor, expressed in three sets of dimensions: the physical space-time domain, the Fourier space-time domain, and the amplitude domain. When combined with *veillametrics* we can measure a sensor's capacity-to-sense, as that capacity-to-sense propagates through space. Physical-Fourier-Amplitude visualizations can be generated for sensors, transducers, signal-processing systems, or physical phenomena, to yeild further insight beyond conventional frequency spectra or sensitivity charts. We examine applications in the sonic and optical modalities.**

***Keywords*—space-time, amplitude domain, high dynamic range (HDR), composited dynamic range (CDR), veillametrics**

I. INTRODUCTION: PFA DOMAIN

We will introduce a new analysis method for signals, and in section II, a new method to quantify sensors—and the act of sensing itself—in a multi-dimensional space.

Signals, whether measured in volts, newtons, pascals, kelvins, or other units, exist over various dynamic ranges, frequency ranges, time ranges and spatial configurations [1], [2]. The physical space-time domain commonly includes three spatial dimensions and time as a fourth dimension. A less-known "amplitude domain" was suggested in [3], with reference to digital photography (mapping brightness of each pixel).

We will introduce a Physical-Fourier-Amplitude (PFA) transformation and domain, allowing a signal to be measured in three sets of dimensions: the physical domains (space-time: x, y, z, t), the Fourier domains (space-time: u, v, w, f), and the amplitude domain (A). We will demonstrate a simplified set of dimensions in this paper. First, to be able to convert to Amplitude-Fourier space, we devised an AF transform, as illustrated in Figs. 1 and 3. Rather than indicating one signal strength at each position or frequency, the AF transform is determined by taking a 2D joint histogram of the spectrogram $\mathcal{F}_{f,\eta}\{s(t)\}$, over values of frequency and intensity. The process, illustrated in Fig. 3, is expressed as:

$$T^{\text{AF}}(A, f)\{s(t)\} = \int_{\{\eta|\mathcal{F}=A\}} d\eta \quad \text{where } \mathcal{F} = \mathcal{F}_{f,\eta}\{s(t)\} \quad (1)$$

A simple way to understand the vertical "A" dimension of this domain is the *range density function* (RDF) [4], which expresses the population of instantaneous values of a time-varying signal. Fig. 2 illustrates. Now, by utilizing the AF transform, we can then create a new *harmonic range density function* (HRDF), to express the population of activity occurring at each *amplitude* for all frequencies. See Fig. 3. The HRDF is computationally-determined through harmonic analysis of the signal, converting by AF transform, and accumulating over each frequency in the spectral domain.

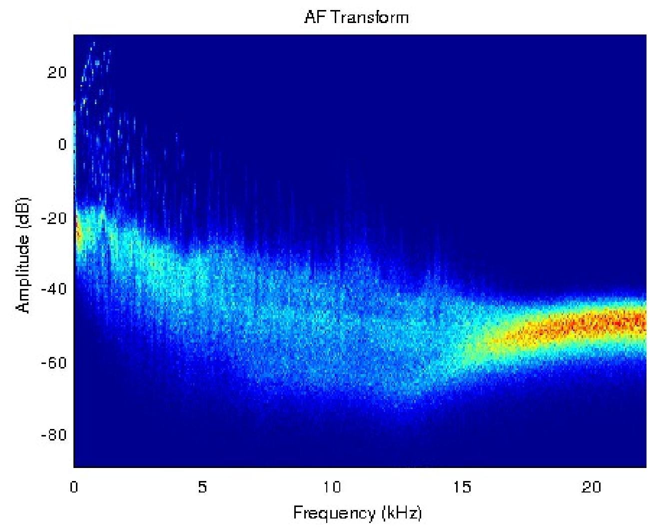

Fig. 1: The AF transform is introduced in order to "sense sensors" and "measure measurement" in this paper. However, in this simple example, it is merely showing a signal: a flute playing a musical scale. The AF-space is vertically similar to the FFT (but contains much more information than the FFT), and horizontally similar to the HRDF (Harmonic Range Density Function). In this example, 18 notes were played on flute in sequence for approximately 1 second each. Since the flute requires more air to be blown as the frequency increases, thus delivering more power for higher notes, this can be observed in the upper-left corner as a series of slanted lines. The notes were played without amplitude modulation, and thus the slanted lines are narrow with each frequency being at a specific amplitude range. Since each note has a relatively short duration (compared to the duration over which the respective frequency is at a much lower amplitude), the activity in the upper-left corner is relatively weak, and much of the information visible is describing the noise-floor (electrical noise and ambient noise). Between 5 and 15 kHz, the amplitude range is quite broad, suggesting this noise varies over time and is mainly acoustic noise, whereas the narrower amplitude range above 15 kHz may be primarily electrical noise at uniform amplitude.

II. SENSING SENSING ITSELF, USING THE PFA DOMAIN

A. Veillametrics: signals in reverse

Veillametrics is a recently-introduced field of study, quantifying "capacity-to-sense" as the capacity-to-sense "emitted" from a sensor propagates spatially, reflects and refracts off of various materials, and attenuates by *veillance degeneracy* [5], [6], [7], [8], [9], The elementary physics concepts: the *veillon*, the *vixel*, and *veillance flux* [5], [6], [7] are metrics of sensory "emissions" through space, outward from a sensor.

Veillametrics measures the capacity to see, observe, or otherwise sense (from French, *veiller*). An intersecting web of veillance field lines and veillance flux, ordinarily hidden in the

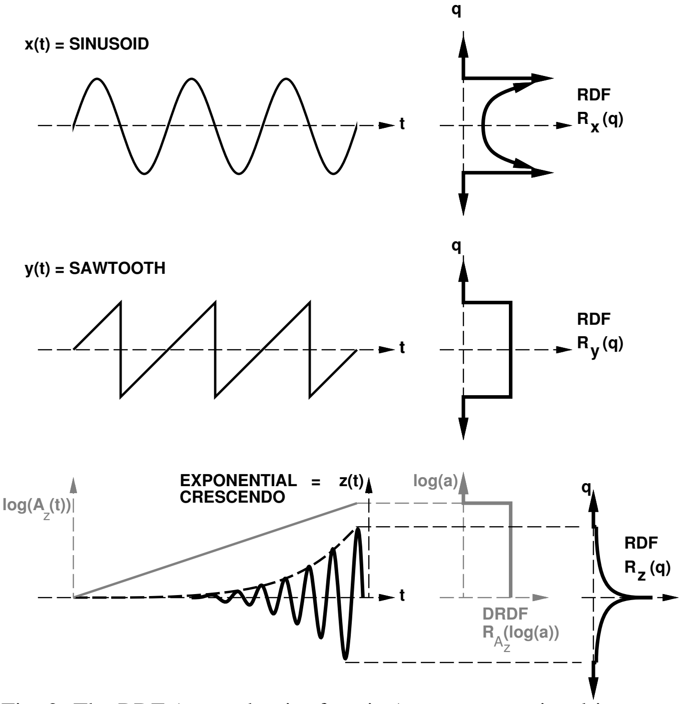

Fig. 2: The RDF (range density function) expresses a signal in terms of its range rather than its domain [4]. The RDF is a deterministic analogue to the PDF (probability density function). In the third example, a sinusoid modulated with an exponential crescendo could be expressed by a hypothetical *dynamic* range density function (DRDF) according to the shape of the crescendo envelope. However, to enable scientific measurements of sensors, we introduce PFA-space, which gives us a new *harmonic* range density function (HRDF).

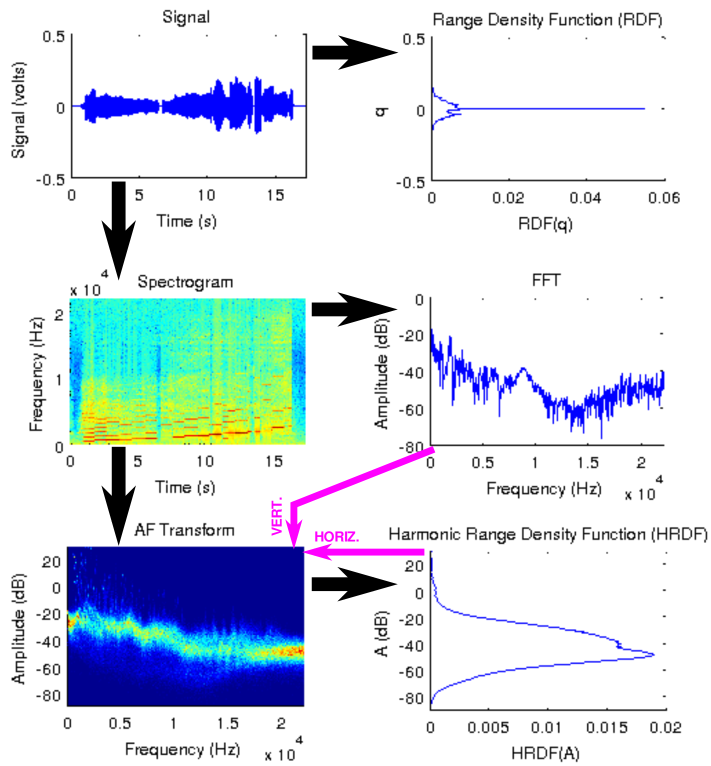

Fig. 3: AF Transform: black arrows indicate the sequence of operations. The AF-space (lower left) is vertically similar to the FFT (but contains much more information than the FFT, since the FFT is single-valued at each frequency), and horizontally similar to the HRDF (Harmonic Range Density Function). The AF transform is illustrated for an ordinary signal (recording of piccolo playing a musical scale), but will be applied in particular to "sense sensors".

world around us, can be measured as they propagate outward from sensors such as cameras [5], [6], audio and temperature sensors [5], and even by the human eyes [9]. Veillance flux can be measured as it moves through space, reflects, refracts, diffuses, concentrates, or creates "veillance shadows" [5], [6].

B. PFA Domain to measure sensory propagation

We now extend this to the amplitude and frequency domain. The domain of a sensory process is thus measured in three sets of dimensions: the amplitude domain (physical or Fourier), the temporal domain (physical or Fourier), and the spatial domain (physical or Fourier). Every sensing process captures phenomena "over" a subset within these dimensions. For example, a microphone might sample over a certain amplitude domain, frequency domain, and spatial domain, with specific precision and resolving power in each of those domains. In particular, while a measured quantity is often thought of as being part of the **range** of a signal (as measured in volts, amperes, newtons, pascals, or kelvins, *etc.*), we instead view a quantity-space as part of the the **domain** of a sensory process. To *measure* this capacity-to-sense, we combine the functions of three devices:

1) Amplitude distribution analyzer [10] (to determine the dynamic range of sensing, and noise floor)
2) Fourier analyzer (determine freq. range of sensing)
3) Veillance flux meter [5], [6] (to determine spatial extent of the sensory zone, and the linear independence of sensor readings throughout space)

With a new function that jointly evaluates the above functions, we can gain new insight into spatio-temporal variation of sensor SNR that we wouldn't otherwise see with these functions individually. PFA-space expresses layered information about *both* strong/weak sensor readings, high/low frequencies, and multiple locations, simultaneously, by being constructed like a histogram, not just filling in one point on a graph for one amplitude at a certain frequency. We express a sensor's capacity-to-sense over a multidimensional domain, S:

$$S = S(\underline{A}, \underline{t}, \vec{X}) \qquad (2)$$

Amplitude $\underline{A}$, temporal $\underline{t}$, and spatial $\vec{X}$ domains can be expressed either in their respective physical or Fourier forms. In this work S was determined by using the 2D joint histogram from the AF transform, using its frequency axis to cover the Fourier $\underline{t}$ domain and amplitude axis to cover the $\underline{A}$ domain. See Fig. 4. Alternatively, the value of S (the *range* of PFA-space) can also express: accuracy, precision, or number of linearly-independent sensor readings affected by activity at that point in space.

This reveals the behavior of sensors which accentuate low and high frequencies differently depending on distance. For example, dynamic microphones accentuate low frequencies in the near-field (the "booming announcer voice"). This can be seen in Fig. 4, where we tested capacity-to-sense at different positions using a moving speaker. The process also characterizes background reflections and noise as part of the sensory

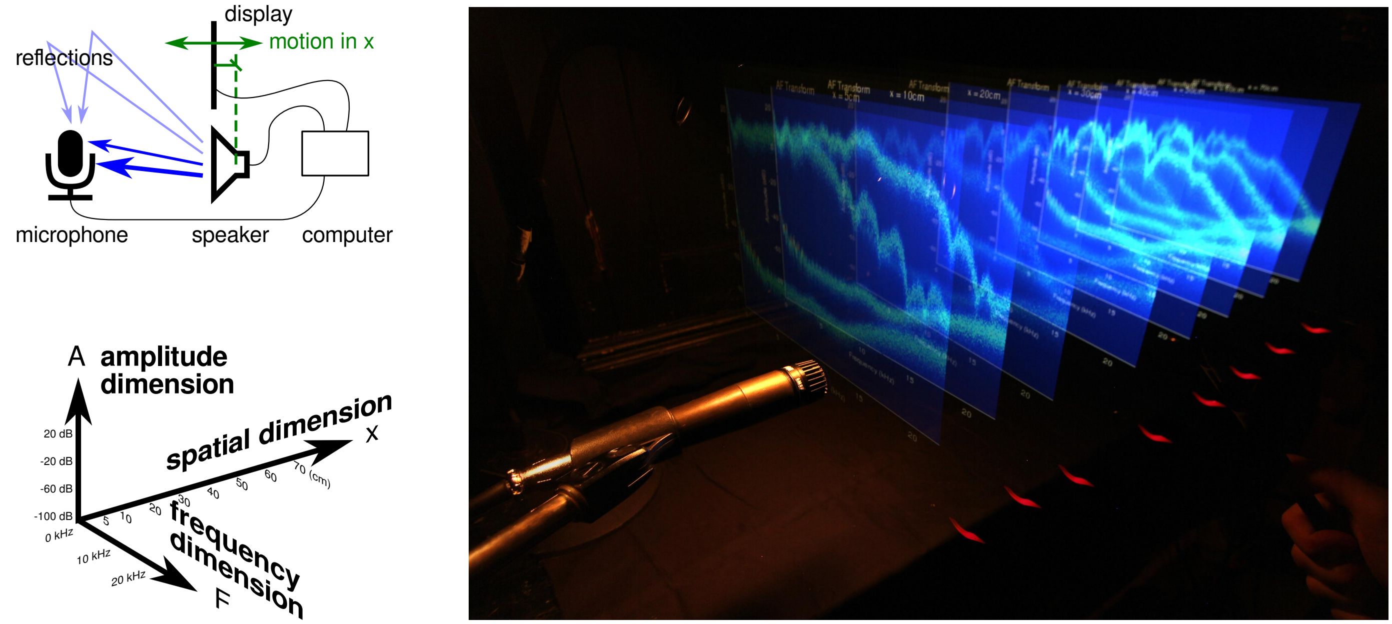

Fig. 4: Physical-Fourier-Amplitude (PFA-Space) measurement of an SM57 microphone. A hyperspace can be formed with the physical spatial dimension(s), amplitude dimension, and frequency (time) dimension, to visualize the capacity-to-sense of the microphone as that capacity-to-sense propagates through space. To form this measurement, a speaker emitting a white-noise signal was positioned at different distances in front of the microphone. The microphone's output signal was AF-transformed for each position. A display of the AF transforms was moved to same positions as the speaker, in this multiple-exposure image. Notice that the AF transform's traces merge together between signal (upper locus) and noise (lower locus), for high frequencies at a distance of 70 cm. At that point in 3D PFA-space (*i.e.* at that spatial point and frequency point), the SNR therefore falls below 1. (Text labels are visible at the top/bottom of each graph, upon close inspection.)

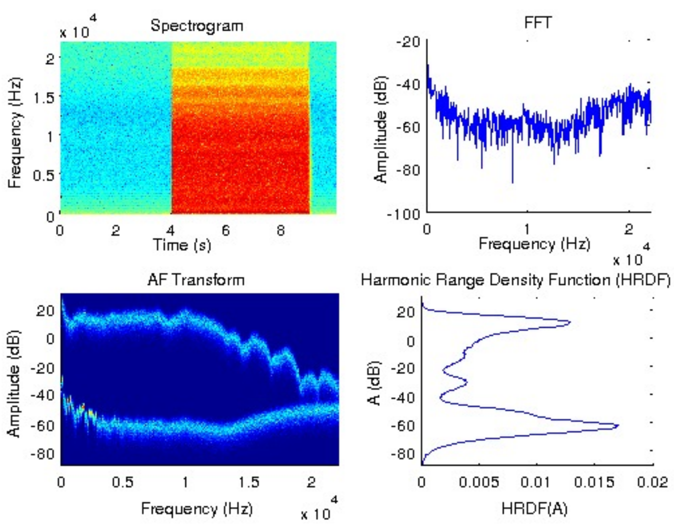

Fig. 5: Measuring the capacity-to-sense of an SM57 microphone. A white-noise signal was emitted 5cm away from the microphone for 5 seconds, as well as 5 seconds of silence (merely background noise). The AF transform expresses more complete information than the FFT: It simultaneously expresses the noise floor (activity at lower locus of points) and the perceived signal (activity at upper locus of points). The space can be further filled by sweeping test signals or stimuli over the $(\underline{A}, \underline{t}, \vec{X})$ dimensions. We can thus build a multidimensional measurement of the sensor's response characteristics. By sensing sensors and measuring measurement, we populate a "meta-sensory space" or "meta-measurement space". In this example, the AF Transform also allows us to see a statistically-expressed dynamic range available at 5cm position, based on SNR above the noise floor.

process. Data can be read off the graphs, to arrive at simplified numerical description of the sensory process, including the noise and background reflections, as in the following table:

Dist.	Dynamic Range @ 200 Hz	Exper. Sens. @ 200Hz	Theoretical data for SM57 model
5cm	>73dB±12dB	$\Delta=+8$dB±6dB	$\Delta=+7$dB
60cm	>41dB±12dB	@ Near-field	@ Near-field

We thus can measure sensors and sensing according to an extramissive framework, as if the capacity-to-sense was "emitted" by the sensor and travels through physical space.

This concept is further illustrated by Fig. 6, where we adapted the PFA-Domain to optical sensing-of-sensing in the human eye. We applied the new analysis algorithms to our previous human eye measurements, expressing the eye's "emissions" of sensory capacity, as that capacity-to-sense propagates through space, also showing AF variation in PFA-domain.

Sensors are typically limited by their fixed, constant, finite capability in PFA-space. Using PFA-space to analyze a sensor's performance, a sensing system could then be improved by strategically filling in PFA-space. Vertically tiling PFA-space in the amplitude domain could be accomplished by composite-dynamic-range (CDR) sampling: combining multiple exposures of a sensory phenomenon (for images [11][12][13] or audio [4]). Thus, a mosaic of sections of PFA-space are joined at different amplitude ranges, to capture dynamic ranges beyond the allowable dynamic range of a single sensor. Similarly, we can tile the PFA-space in the frequency domain by using frequency-range compositing, combining complementary sensors whose PFA-domains are strategically chosen.

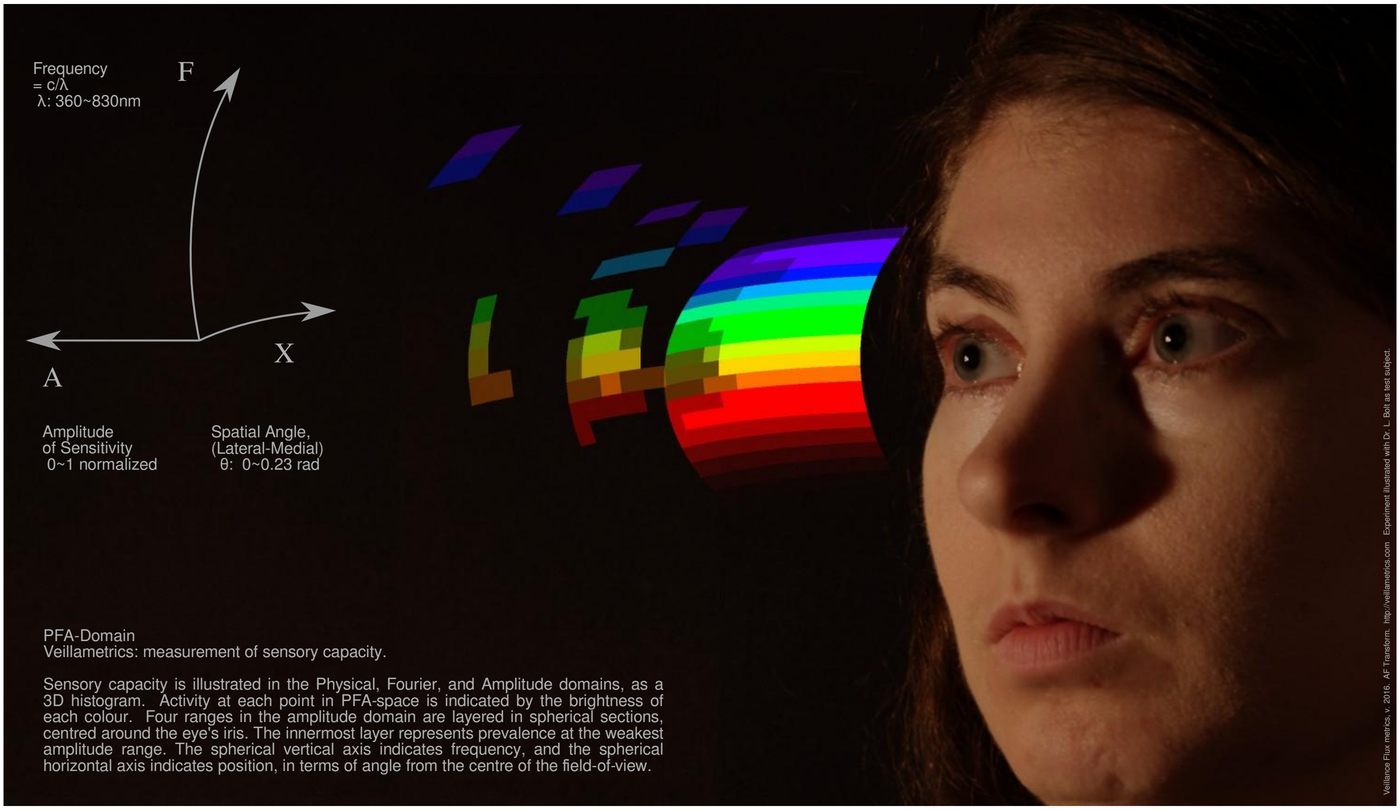

Fig. 6: Physical-Fourier-Amplitude (PFA-space) measurement of the human eye. We combine our previous "sensing-of-sensing" (veillance flux) measurements of the spatial sensory "emissions" of human eyesight [9]. We incorporated spectral colour sensitivity, as measured in [14], [15], and applied the AF-transform. The result is a 3-dimensional visualization of light sensitivity in the amplitude, frequency, and position dimensions. PFA-space is a 3D sensitivity histogram, in this example, while PFA-space can alternatively express a measurement certainty function [11], or a measurement error, at each $(\underline{A}, \underline{t}, \vec{X})$ point. A coarse resolution here illustrates the principle of the 3D histogram, and a fine resolution can also be used similarly to Figs. 1, 3, 4, and 5. In this figure, the computer-generated data was aligned and composited with the individual's photograph in post-processing. Activity at each point in PFA-space is indicated by the alpha channel (brightness) of the given colour. The innermost layer represents activity at the weakest amplitude range. Four layers in the amplitude domain are layered in spherical sections, centred around the eye's iris. Each sphere's vertical axis indicates frequency (wavelength) of sensed light. Each sphere's horizontal axis indicates position, in terms of angle from the centre of the field-of-view. Colours correspond to the physical wavelength being measured.

III. Conclusion

While an ordinary microphone sensitivity curve (a line *v.s.* frequency and/or space) only expresses one amplitude value at each point on the curve, we now have a 3+ dimensional analysis, with amplitude as a *domain*. The AF Transform expresses more complete information than the Fourier transform (shown in Figs. 3, 5). We can now measure a signal (or sensor itself) according to a more expressive and informative domain – over amplitude, physical and Fourier dimensions.

`http://veillametrics.com` — Sample code available.

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